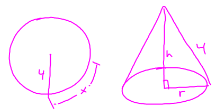


$C = 8\pi - x$

$r_{cone} = \dfrac{8\pi - x}{2\pi}$ or $4 - \dfrac{x}{2\pi}$

$h_{cone} = \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)^2}$

$V_{cone} = \frac{\pi}{3} r^2 h$

$$\frac{\pi\left(4-\frac{x}{2\pi}\right)^2 \cdot \sqrt{16-\left(4-\frac{x}{2\pi}\right)^2}}{3} \longrightarrow \frac{\pi\left(16-\frac{8x}{2\pi}+\frac{x^2}{4\pi^2}\right)\cdot\sqrt{16-\left(16-\frac{8x}{2\pi}+\frac{x^2}{4\pi^2}\right)}}{3}$$

$$\frac{\left(16\pi - 4x + \frac{x^2}{4\pi}\right)\cdot\sqrt{\frac{4x}{\pi} - \frac{x^2}{4\pi^2}}}{3} \Rightarrow \frac{\frac{64\pi^2 - 16\pi x + x^2}{4\pi}\cdot\sqrt{\frac{16\pi x - x^2}{4\pi^2}}}{3}$$

$$\frac{\frac{64\pi^2 - 16\pi x + x^2}{4\pi}\cdot\frac{\sqrt{16\pi x - x^2}}{2\pi}}{3} \Rightarrow \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$$

$$\boxed{\text{Volume} = \frac{(8\pi-x)^2(16\pi x - x^2)^{\frac{1}{2}}}{24\pi^2}}$$

$$\frac{dV}{dx} = \frac{1}{24\pi^2}\left[(8\pi-x)^2\cdot\frac{1}{2}(16\pi x - x^2)^{-\frac{1}{2}}\cdot(16\pi - 2x) + (16\pi x - x^2)^{\frac{1}{2}}\cdot 2(8\pi - x)(-1)\right]$$

$$0 = \frac{1}{2}(8\pi - x)^2(16\pi x - x^2)^{-\frac{1}{2}}(16\pi - 2x) - 2(16\pi x - x^2)^{\frac{1}{2}}(8\pi - x)$$

easy enough - right?

$$\frac{\frac{1}{2}(8\pi-x)^{\cancel{2}}(16\pi x - x^2)^{-\frac{1}{2}}(16\pi - 2x)}{(8\pi - x)} = \frac{2(16\pi x - x^2)^{\frac{1}{2}}(8\pi - x)}{(8\pi - x)}$$

$$2\cdot\frac{1}{2}(8\pi - x)(16\pi x - x^2)^{-\frac{1}{2}}(16\pi - 2x) = 2(16\pi x - x^2)^{\frac{1}{2}}$$

on bottom, mult both sides

$$(8\pi - x)(16\pi - 2x) = 4(16\pi x - x^2)$$

expand distribute

$$128\pi^2 - 16\pi x - 16\pi x + 2x^2 = 64\pi x - 4x^2$$

$$128\pi^2 - 32\pi x + 2x^2 = 64\pi x - 4x^2$$

$-2x^2$ $-2x^2$

$+32\pi x$ $+32\pi x$

$-128\pi^2$

$$0 = -6x^2 + 96\pi x - 128\pi^2$$

Quadratisize

$$x = \frac{-96\pi \pm \sqrt{(96\pi)^2 - 4(-6)(-128\pi^2)}}{-12}$$

$x \approx 4.612$ or 45.654

extraneous solution

Main Ideas!

Critical Points

- where $f'(x) = 0$ or is undefined
- $f(x)$ must exist at that point

Extrema of $f(x)$

- Absolute min/max $\rightarrow$ only on a closed interval
- local (relative) min/max $\rightarrow$ could be on open interval
- can occur when $f'(x) = 0$, is undefined, or at endpoints (closed interval)

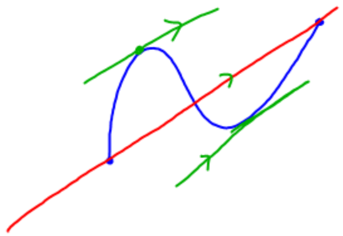
Increasing/decreasing $f(x)$

- If $f'(x) > 0$ at each point of (a, b), then $f(x)$ increases on $[a, b]$
- If $f'(x) < 0$ at each point of (a, b), then $f(x)$ decreases on $[a, b]$
- If $f'(x) = 0$ at each point of (a, b), then $f(x)$ is constant on $[a, b]$

Mean Value Thrm

- If $f(x)$ is continuous at every point of a closed interval $[a, b]$, and differentiable at at every point (a, b), then there is at least one point c in (a, b) such that

$$\underbrace{f'(c)}_{\text{tangent}} = \underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{secant line}}$$



First Derivative Test

- If c is a critical number of $f(x)$
  1. If $f'(x)$ changes from positive to negative at c, then $f(x)$ has a relative max. at c.
  2. If $f'(x)$ changes from negative to positive at c, then $f(x)$ has a relative min. at c.

Concavity

- If $f''(x)$ is positive, then $f(x)$ concave up $\cup$
- If $f''(x)$ is negative, then $f(x)$ concave down $\cap$

Point of inflection

- A point where $f(x)$ has a tangent line and where the concavity changes.

Second derivative test

- If $f'(c)=0$ (have critical pt.) and $f''(c)<0$, then $f(x)$ has a local max at c.

- If $f'(c)=0$ and $f''(c)>0$, then $f(x)$ has a local min. at c

- If $f''(c)=0$, the test fails

Do these analytically

(1) Find the extrema of $f(x) = 2x - 3x^{2/3}$ on $[-1, 3]$

(2) Find the extrema of $f(x) = 2\sin x - \cos 2x$ on $[0, 2\pi]$

(3) Find points of inflection, extrema, and discuss the concavity of the graph of $f(x) = x^4 - 4x^3$

(4) Show how to sketch a "good" graph of
$y = x^4 - 12x^3 + 48x^2 - 64x$

(5) Analyze graph of $f(x) = \dfrac{\cos x}{1 + \sin x}$

Answers

① $f'(x) = 2 - \frac{2}{x^{1/3}} = 2\left(\frac{x^{1/3}-1}{x^{1/3}}\right)$

Zeros at $x=1$
undef at $x=0$

x	-1	0	1	3
$f(x)$	-5	0	-1	-0.24
	min (abs)	max (abs)	local min	

	$+$		$-$		$+$
		0		1	
		max		min	

② $f'(x) = 2\cos x + 2\sin 2x = 0$

$2\cos x + 4\cos x \sin x = 0$

$(2\cos x)(1 + 2\sin x) = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $\frac{7\pi}{6}, \frac{11\pi}{6}$

$+$		$-$		$+$		$-$		$+$
	$\frac{\pi}{2}$		$\frac{7\pi}{6}$		$\frac{3\pi}{2}$		$\frac{11\pi}{6}$	
	max		min		max		min	
	(abs)		(abs)				(abs)	

③ $f'(x) = 4x^3 - 12x^2$ $\qquad$ $f''(x) = 12x^2 - 24x = 12x(x-2)$

$0 = 4x^2(x-3)$ $\qquad$ $0 = 12x(x-2)$

$x = 0, 3$ $\qquad$ $x = 0, 2$

$-$ $\quad$ $-$ $\quad$ $+$ $\qquad$ $+$ $\quad$ $-$ $\quad$ $+$

0 $\quad$ 3 $\qquad$ 0 $\quad$ 2

min $\qquad$ inflect $\quad$ inflect.

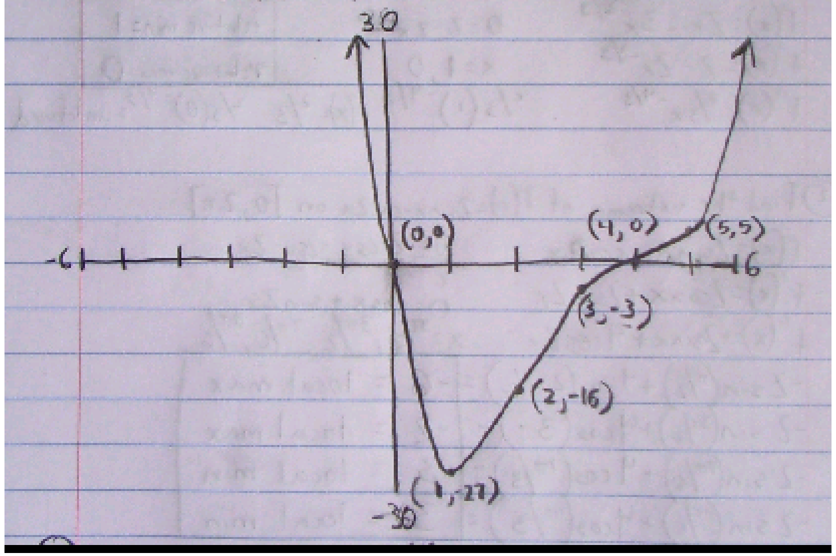
④ $f(x) = x(x-4)^3$ $\qquad$ $f'(x) = 4(x-1)(x-4)^2$ $\qquad$ $f''(x) = 12(x-4)(x-2)$

x-int: $(0,0)(4,0)$, y-int: $(0,0)$, End behavior both sides $\uparrow$, critical #'s $x=1$, $x=4$

min

inflect. pts $x = 2, 4$

④ Sketch a graph of $y = x^4 - 12x^3 + 48x^2 - 64x$

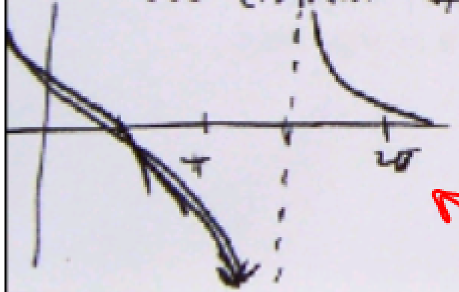


5 $f'(x) = -\frac{1}{1+\sin x}$ $f''(x) = \frac{\cos x}{(1+\sin x)^2}$

Capture a user defined rectangle.

x-int: $(\frac{\pi}{2}, 0)$ y-int $(0,1)$ vert. asymp. $x = -\frac{\pi}{2}, \frac{3\pi}{2}$

No critical #'s inflect. point $x = \frac{\pi}{2}$



Not a good graph

Analyze the graph of $y = \sin x - \frac{1}{18}\sin 3x$, $0 \le x \le 2\pi$

Do it analytically (extrema, inflection pts, concavity, etc.)

This is the last question of the test. You should bring it in on Wed.

$\boxed{\text{HW}}$

- study for test
- Do this problem